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Interactive Control of Turbulent Boundary Layers: A Futuristic Overview

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I. Introduction

THE ability to manipulate a flowfield to effect a desired change is of immense technological importance. As defined by Flatt,[46] the term boundary-layer control includes any mechanism or process through which the boundary layer of a fluid flow is caused to behave differently than it normally would were the flow developing naturally along a smooth straight surface. The control device could be passive, requiring no auxiliary power, or active, requiring energy expenditure. Interactive (or reactive) control is a special class of active control where the control input is continuously adjusted based on measurements of some kind (Wilkinson[148]). The control loop in this case could either be an open one or a closed (feedback) loop.

An external wall-bounded flow, such as that developing on the exterior surface of an aircraft or a submarine, could be manipulated to achieve transition delay, separation postponement, lift enhancement, drag reduction, turbulence augmentation, or noise suppression. These objectives are not necessarily mutually exclusive. For example, by maintaining as much of a boundary layer in the laminar state as possible, the skin-friction drag and the flow-generated noise are reduced. However, a turbulent boundary layer is in general more resistant to separation than a laminar one. By preventing separation, lift is enhanced and the form drag is reduced. An ideal method of control that is simple, is inexpensive to build and to operate, and does not have any tradeoff does not exist, and the skilled engineer has to make continuous compromises to achieve a particular goal.

Numerous methods of flow control have already been successfully implemented in practical engineering devices. Such techniques have been reviewed by, among others, Bushnell,[20] Wilkinson et al.,[151] Bushnell and McGinley,[22] Gad-el-Hak,[50] Bushnell and Hefner,[21] Fiedler and Fernholz,[45] and Gad-el-Hak and Bushnell.[55] Yet, limitations exist for some familiar control techniques when applied to specific situations. For example, in attempting to reduce the drag or enhance the lift of a body having a turbulent boundary layer using global suction, the penalty associated with the control device often exceeds the saving derived from its use. What is needed is a way to reduce this penalty to achieve a more efficient control. The main objective of this article is to illustrate a possible scenario by which efficient control could be realized. As will be argued in the following presentation, future systems for control of turbulent flows in general and turbulent boundary layers in particular could greatly benefit from the merging of the science of chaos control and the technology of microfabrication. Such systems are envisaged as consisting of a large number of intelligent, interactive wall sensors and actuators arranged in a checkerboard pattern and targeted toward specific organized structures that occur randomly within the boundary layer.

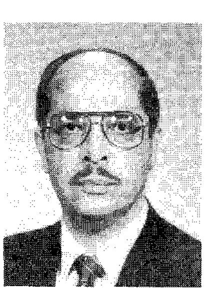
The present paper is organized into eight sections. A particular example of a classical control system, suction, is described in the following section. In Sec. III, the different hierarchies of coherent structures that dominate a turbulent boundary layer are briefly recalled. Targeted control and the selective suction concept are described in Sec. IV. Sections V and VI introduce the reader to some of the exciting new developments in the fields of chaos control and microfabrication, respectively. In Sec. VII, a possible scenario for using those developments to control a turbulent flow is proposed. Finally, brief concluding remarks are given in Sec. VIII.

II. Classical Control Systems

A. Mean-Velocity Modifiers

A viscous fluid that is initially irrotational will acquire vorticity when an obstacle is passed through the fluid. This vorticity controls the nature and structure of the boundary-layer flow. For an incompressible, wall-bounded flow, the flux of spanwise vorticity at the wall, and hence whether the surface is a sink or a source of vorticity, is affected by the wall motion (e.g., in the case of a compliant coating), transpiration (suction or injection), streamwise pressure gradient, wall curvature, and normal viscosity gradient near the wall (caused by, for example, heating/cooling of the wall or introduction of a shear-thinning/shear-thickening additive into the boundary layer). These alterations separately or collectively control the shape of the mean velocity profile, which in turn determines the skin friction at the wall, the boundary layer's ability to resist transition and separation, and the intensity of turbulence and its structure.

For illustration purposes, we focus on global wall suction as a generic control tool. The arguments presented here and in subsequent sections are equally valid for other global control techniques, such as geometry modification (body shaping), surface heating/cooling, etc. As will be illustrated later, transpiration pro-



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vides a good example of a single control technique that is used to achieve a variety of goals. Suction leads to a fuller velocity profile (vorticity flux away from the wall) and can, thus, be employed to delay laminar-to-turbulent transition, postpone separation, achieve an asymptotic turbulent boundary layer (i.e., constant momentum thickness), or relaminarize an already turbulent flow. Unfortunately, global suction cannot be used to reduce the skin-friction drag in a turbulent boundary layer. The amount of suction required to inhibit boundary-layer growth is too large to effect a net drag reduction. This is a good representation of a situation where the penalty associated with a control device might exceed the saving derived from its use.

B. Governing Equations

Before detailing how global suction affects the flow, it might be instructive to recall two special versions of the equations of motion. First, for a steady, incompressible flow around a two-dimensional or axisymmetric surface of small curvature, the continuity and streamwise momentum equations can be integrated in the normal direction to yield the von Kármán integral relation:

$$C_f = 2\left(\frac{d\theta}{dx}\right) - 2\left(\frac{\upsilon_w}{U_0}\right) + 2\theta\left\{\left(2 + \left[\frac{\delta^*}{\theta}\right]\right)\left(\frac{1}{U_0}\right)\left(\frac{dU_0}{dx}\right) + \left(\frac{1}{R}\right)\left(\frac{dR}{dx}\right)\right\} \tag{1}$$

where C_f is the local skin-friction coefficient, δ^* and θ are the displacement and momentum thicknesses, respectively, υ_w is the normal velocity of fluid injected through the surface (positive for injection and negative for suction), U_0 is the velocity outside the boundary layer, and R is the radius of curvature of the wall. Equation (1) is valid for both laminar and turbulent boundary layers. In the latter case, time-averaged flow quantities are used throughout the equation.

For a zero-pressure-gradient flat plate with suction, the previous equation simplifies to

$$C_f = 2\left(\frac{d\theta}{dx}\right) + 2C_q \tag{2}$$

where C_q is the suction coefficient ($\equiv |\upsilon_w|/U_0$).

The second useful equation is obtained from the instantaneous streamwise momentum balance as the wall is approached ($y = 0$). For a noncompliant wall, this equation reads

$$\upsilon_w\left[\frac{\partial U}{\partial y}\right]_0 + \frac{1}{\rho}\left[\frac{\partial P}{\partial x}\right]_0 - \left[\frac{\partial \nu}{\partial y}\right]_0\left[\frac{\partial U}{\partial y}\right]_0 = \nu\left[\frac{\partial^2 U}{\partial y^2}\right] \tag{3}$$

where the subscript $[\ ]_0$ indicates flow quantities computed at the wall, x and y are the streamwise and normal coordinates, respectively, U is the instantaneous streamwise velocity component, υ_w is the normal velocity at the wall, ρ is the (constant) density, P is the instantaneous pressure, and ν is the kinematic viscosity that in general varies with space and time as a result of surface heating/cooling, film boiling, cavitation, sublimation, chemical reaction, wall injection of higher/lower viscosity fluid, or the presence of a shear-thinning/thickening additive.

Suction (or downward wall displacement), favorable pressure gradient, or lower near-wall viscosity leads to a more negative curvature of the velocity profile at the wall (fuller profile). Equation (3) could be used to compute the magnitude of any of those modulations necessary to achieve a given degree of boundary-layer stabilization.

C. Suction

Small amounts of fluid withdrawn from the near-wall region of a boundary layer change the curvature of the velocity profile at the wall and can dramatically alter the stability characteristics of the flow. Concurrently, suction inhibits the growth of the boundary layer, so that the critical Reynolds number (the lowest Reynolds number at which linear Tollmien-Schlichting waves would grow in a particular laminar flow) based on thickness may never be reached. Although laminar flow can be maintained to extremely high Reynolds numbers provided enough fluid is sucked away, the goal is to accomplish transition delay with the minimum suction flow rate. This will reduce not only the power necessary to drive the suction pump but also the momentum loss due to the additional freestream fluid entrained into the boundary layer as a result of withdrawing fluid from the wall. That momentum loss is, of course, manifested as an increase in the skin-friction drag.

The case of uniform suction from a flat plate at zero incidence is an exact solution of the Navier-Stokes equation. The asymptotic velocity profile in the viscous region is exponential and has a negative curvature at the wall. The displacement thickness has the *constant* value $\delta^* = \nu/|\upsilon_w|$, where ν is the kinematic viscosity and $|\upsilon_w|$ is the absolute value of the normal velocity at the wall. In this case, the familiar von Kármán integral equation reads as follows: $C_f = 2C_q$. Bussmann and Münz[23] computed the critical Reynolds number for the asymptotic suction profile to be $\boldsymbol{Re}_{\delta^*} \equiv U_\infty\delta^*/\nu = 7 \times 10^4$. From the value of δ^* given earlier, the flow is stable to all small disturbances if $C_q \equiv |\upsilon_w|/U_0 > 1.4 \times 10^{-5}$. The amplification rate of unstable disturbances for the asymptotic profile is an order of magnitude less than that for the Blasius boundary layer (Pretsch[115]). This treatment ignores the development distance from the leading edge needed to reach the asymptotic state. When this is included in the computation, a higher C_q (1.18×10^{-4}) is required to insure stability (Iglisch[68] and Ulrich[143]).

Suction can also be used to postpone separation. Prandtl[112] applied suction through a spanwise slit on one side of a circular cylinder. His flow visualization photographs convincingly showed that the boundary layer adhered to the suction side of the cylinder over a considerably larger portion of its surface. By removing the decelerated fluid particles in the near-wall region, the velocity gradient at the wall is increased, the curvature of the velocity profile near the surface becomes more negative, and separation is avoided. Prandtl[113] used the momentum integral equation to make a simple estimate of the required suction coefficient to prevent laminar separation from the cylinder's surface: $C_q = 4.36Re^{-0.5}$, where Re is the Reynolds number based on the cylinder diameter and the freestream velocity.

Several researchers have used similar approximate methods to calculate the laminar boundary layer on a body of arbitrary shape with arbitrary suction distribution (e.g., see Chang[25,26] and Schlichting[121]). A particularly simple calculation is due to Truckenbrodt.[142] He reduces the problem to solving a first-order ordinary differential equation. As an example, for a symmetrical Zhukovskii's airfoil with uniform suction, Truckenbrodt predicts a suction coefficient just sufficient to prevent separation of $C_q = 1.12Re^{-0.5}$, where Re is the Reynolds number based on the airfoil chord and the freestream velocity.

For turbulent boundary layers, semi-empirical methods of calculation are inevitably used due to the closure problem. Suction coefficients in the range of $C_q = 0.002$–0.004 are sufficient to prevent separation on a typical airfoil (Schlichting[121]). Optimally, the suction should be concentrated on the low-pressure side of the airfoil just a short distance behind the nose where, at large angles of attack, the largest local adverse pressure gradient occurs.

In a turbulent wall-bounded flow, the results of Eléna[36,37] and more recently of Antonia et al.[3] indicate that suction causes an appreciable stabilization of the low-speed streaks in the near-wall region. The maximum turbulence level at $y^+ \approx 13$ drops from 15 to 12% as C_q varies from 0 to 0.003. More dramatically, the tangential Reynolds stress near the wall drops by a factor of 2 for the same variation of C_q. The dissipation length scale near the wall increases by 40% and the integral length scale by 25% with the suction.

The suction rate necessary for establishing an asymptotic turbulent boundary layer independent of streamwise coordinate ($d\theta/dx = 0$) is much lower than the rate required for relaminarization ($C_q \approx 0.01$) but still not low enough to yield net drag reduction. For $Re = \mathcal{O}[10^6]$, Favre et al.,[43] Rotta,[120] and Verollet et al.,[144] among others, report an asymptotic suction coefficient of $C_q \approx 0.003$. For a zero-pressure-gradient boundary layer on a flat plate, the corresponding skin-friction coefficient is $C_f = 2C_q = 0.006$, indicating higher skin friction than if no suction was applied. To achieve a net skin-friction reduction with suction, the process must be further optimized. One way to achieve that is to target the suction toward a particular organized structure within the boundary layer and not to use it globally as in classical control schemes. This point will be revisited in Sec. IV.

III. Coherent Structures

The discussion in Sec. II indicates that achieving a particular control goal is always possible. The challenge is reaching that goal with a penalty that could be tolerated. Suction, for example, would lead to a net drag reduction, if only we could reduce the suction coefficient necessary for establishing an asymptotic turbulent boundary layer to below one-half of the unperturbed skin-friction coefficient. A more efficient way of using suction, or any other global control method, is to aim at a particular coherent structure within the turbulent boundary layer. Before discussing this *selective suction* idea, we briefly describe in this section the different hierarchy of organized structures in a wall-bounded flow.

The classical view that turbulence is essentially a stochastic phenomenon having a randomly fluctuating velocity field superimposed on a well-defined mean has been changed in the last few decades by the realization that the transport properties of all turbulent shear flows are dominated by *quasiperiodic*, large-scale vortical motions (Laufer,[87] Cantwell,[24] Fiedler,[44] and Robinson[118]). What follows is a brief description of the multiplicity of coherent structures that have been identified in turbulent boundary layers, mostly through low-Reynolds-number experiments (see Gad-el-Hak and Bandyopadhyay[52] for a discussion of Reynolds number effects).

In a boundary layer, the turbulence production process is dominated by three kinds of quasiperiodic eddies: the large outer structures, the intermediate Falco eddies, and the near-wall eddies. The large, three-dimensional structures scale with the boundary-layer thickness δ and extend across the entire layer (Kovasznay et al.[80] and Blackwelder and Kovasznay[17]). These eddies control the dynamics of the boundary layer in the outer region, such as entrainment, turbulence production, etc. They appear randomly in space and time and seem to be, at least for moderate Reynolds numbers, the residue of the transitional Emmons spots (Zilberman et al.[155] and Gad-el-Hak et al.[57]). The Falco eddies are also highly coherent and three dimensional. Falco[38,39] named them typical eddies because they appear in wakes, jets, Emmons spots, grid-generated turbulence, and boundary layers in zero, favorable, and adverse pressure gradients. They have an intermediate scale of about $100\nu/u^*$ (100 wall units; u^* is the friction velocity and ν/u^* is the viscous length scale). The Falco eddies appear to be an important link between the large structures and the near-wall events.

The third kind of eddies exists in the near-wall region ($0 < y < 100\nu/u^*$) where the Reynolds stress is produced in a very intermittent fashion. Half of the total production of turbulence kinetic energy ($-\overline{u\upsilon}\, \partial \bar{U}/\partial y$) takes place near the wall in the first 5% of the boundary layer at typical laboratory Reynolds numbers (smaller fraction at higher Reynolds numbers), and the dominant sequence of eddy motions there are collectively termed the bursting phenomenon. This dynamically significant process was reviewed by Willmarth[152] and Blackwelder[15] and most recently by Robinson.[108] Qualitatively, the process, according to at least one school of thought, begins with elongated, counter-rotating, streamwise vortices having diameters of approximately $40\nu/u^*$. The vortices exist in a strong shear and induce low- and high-speed regions between them. The vortices and the accompanying eddy structures occur randomly in space and time. However, their appearance is regular enough that an average spanwise wavelength of approximately 80–$100\nu/u^*$ has been identified by Kline et al.[77] and others. Kline et al.[77] also observed that the low-speed regions grow downstream and develop inflectional $U(y)$ profiles. At approximately the same time, the interface between the low- and high-speed fluid begins to oscillate, apparently signaling the onset of a secondary instability. The low-speed region lifts up away from the wall as the oscillation amplitude increases, and then the flow rapidly breaks down into a completely chaotic motion. Since this latter process occurs on a very short time scale, Kline et al. called it a *burst.* Corino and Brodkey[31] showed that the low-speed regions are quite narrow, i.e., $z = 20\nu/u^*$, and may also have significant shear in the spanwise direction. Virtually all of the net production of turbulence kinetic energy in the near-wall region occurs during these bursts.

Considerably more has been learned about the bursting process during the last decade. For example, Falco[40,41] has shown that when a typical eddy, which may be formed in part by ejected wall-layer fluid, moves over the wall, it induces a high $u\upsilon$ sweep (positive u and negative υ). The wall region is continuously bombarded by *pockets* of high-speed fluid originating in the logarithmic and possibly the outer layers of the flow. These pockets tend to promote and/or enhance the inflectional velocity profiles by increasing the instantaneous shear leading to a more rapidly growing instability. Blackwelder and Haritonidis[16] have shown convincingly that the frequency of occurrence of these events scales with the viscous parameters consistent with the usual boundary-layer scaling arguments. An excellent review of the dynamics of turbulent boundary layers has recently been provided by Sreenivasan.[136] More information about coherent structures in high-Reynolds-number boundary layers is given by Gad-el-Hak and Bandyopadhyay.[52]

IV. Targeted Control

Targeted control implies sensing and reacting to a particular quasiperiodic structure in the boundary layer. The wall seems to be the logical place for such interactive control, because of the relative ease of placing something in there and the sensitivity of the flow in general to surface perturbations. According to Wilkinson,[133] there are very few actual experiments that use embedded wall sensors to initiate a surface actuator response (Alshamani et al.,[1] Wilkinson and Balasubramanian,[149] Nosenchuck and Lynch,[103] and Breuer et al.[19]). This three-year-old assessment is fast changing, however, with the introduction of microfabrication technology that has the potential for producing small, inexpensive, programmable sensor/actuator chips. Witness the more recent interactive control attempts by Kwong and Dowling,[81] Reynolds,[117] Jacobs et al.,[69] Jacobson and Reynolds,[70,71] and Fan et al.[42] The last two references even consider the use of self-learning neural networks for increased computational speeds and efficiency.

In the latter portion of this section, a general discussion of the control goal of achieving drag reduction is followed by a review of the selective suction technique. This will illustrate the potential benefits of utilizing the new science of chaos control and emerging technology of microfabrication, which are, respectively, discussed in Secs. V and VI.

A. Drag Reduction

Techniques to reduce pressure drag are more well established than turbulence skin-friction reduction techniques (Gad-el-Hak[50] and Gad-el-Hak and Bushnell[55]). Streamlining and other methods to postpone separation can eliminate most of the pressure drag. The wave and induced drag contributions to the pressure drag can also be reduced by geometric design. The skin friction constitutes about 50, 90, and 100% of the total drag on commercial aircraft, underwater vehicles, and pipelines, respectively.

For the purpose of reducing skin-friction drag in a wall-bounded flow, three flow regimes are identified. First, for $Re_x < 10^6$, the flow is laminar and the associated low skin friction may be lowered further by reducing the near-wall momentum. Adverse pressure gradient, blowing, and surface heating/cooling (in air or water, respectively) could lower the skin friction but increase the risk of transition and separation. Second, for $10^6 < Re_x < 4 \times 10^7$, active and passive methods to delay transition could be used, thus avoiding the much higher turbulence drag. These techniques have been reviewed by, among others, Gad-el-Hak.[50,51] Third, at the Reynolds number encountered after the first few meters of a fuselage or a submarine ($Re_x > 4 \times 10^7$), methods to reduce the large skin friction associated with turbulent flows are sought. Most of the current research effort concerns reduction of skin-friction drag for turbulent boundary layers. This task is, however, far from trivial.

Techniques to reduce the skin friction of a turbulent flow are classified in the following five categories: reduction of near-wall momentum, introduction of foreign substance, geometrical modification, relaminarization, and synergism. The second category leads to the most impressive results. Introduction of small concentration of polymers, surfactants, particles, or fibers into a turbulent boundary layer leads to a reduction in the skin-friction coefficient of as much as 80%. Among the practical considerations requiring further study are the cost of the additive, methods of delivering it to the boundary layer, potential for recovering and recycling, degradation, and the portion of the payload that has to be displaced to make room for the additive.

Recently introduced techniques mostly fall under the aforementioned third category and seem to offer more modest net drag reduction. These methods are, however, still in the research stage and include riblets (~8%), large eddy breakup (LEBU) devices (~20%), and convex surfaces (~20%). [There is currently strong evidence that LEBUs would not yield any net saving at the higher Reynolds numbers encountered in field applications (Anders[2]).] Potential improvement in these and other methods will perhaps involve combining more than one technique aiming at achieving a favorable effect that is greater than the sum. Because of its obvious difficulties, synergism has not been a very popular research area in the past but deserves future attention.

Successful techniques to reduce the skin friction in a turbulent flow, such as polymers, particles, and riblets, seem to act indirectly through local interaction with discrete turbulent structures, particularly small-scale eddies, within the flow. Common characteristics of all of these methods are increased losses in the near-wall region, thickening of the buffer layer, and lowered production of Reynolds shear stress (Bandyopadhyay[8]). Methods that act directly on the mean flow, such as suction, lowering of near-wall viscosity, or favorable pressure gradient, also lead to inhibition of Reynolds stress. However, skin friction is increased when any of these latter techniques is applied globally.

Could these seemingly inefficient techniques, e.g., global suction, be optimized to reduce their associated penalty? It appears that the more successful drag-reduction methods, e.g., polymers, act selectively on particular scales of motion and are thought to be associated with stabilization of the secondary instabilities. One asks, what would become of wall turbulence if specific coherent structures are to be targeted, by the operator through an interactive control scheme, for modification? The organized structures are instantaneously identifiable, quasiperiodic motions. Bursting events, for example, are both intermittent and random in space as well as time. If such structures are nonintrusively detected and altered, net performance gain might be achieved. It seems clear, however, that temporal phasing as well as spatial selectivity would be required to achieve proper control targeted toward random events.

B. Selective Suction

A noninteractive version of the preceding idea is the *selective suction technique* that combines suction to achieve an asymptotic turbulent boundary layer and longitudinal riblets to fix the location of low-speed streaks. Although far from indicating net drag reduction, the available results are encouraging, and further optimization is needed. When implemented via an array of interactive control loops, the selective suction method is potentially capable of skin-friction reduction that approaches 60%.

The genesis of the selective suction concept can be found in the papers by Gad-el-Hak and Blackwelder[53,54] and the patent by the same authors (U.S. Patent number 4,932,612). Gad-el-Hak and Blackwelder suggest that one possible means of optimizing the suction rate is to be able to identify where a low-speed streak is presently located and apply a small amount of suction under it. Assuming that the production of turbulence kinetic energy is due to the instability of an inflectional $U(y)$ velocity profile, one needs to remove only enough fluid so that the inflectional nature of the profile is alleviated. An alternative technique that could conceivably reduce the Reynolds stress is to inject fluid selectively under the high-speed regions. The immediate effect of normal injection would be to decrease the viscous shear at the wall resulting in less drag. In addition, the velocity profiles in the spanwise direction $U(z)$ would have a smaller shear $\partial U/\partial z$ because the suction/injection would create a more uniform flow. Since Swearingen and Blackwelder[137] have found that inflectional $U(z)$ profiles occur as often as inflection points are observed in $U(y)$ profiles, suction under the low-speed streaks and/or injection under the high-speed regions would decrease this shear and hence the resulting instability. In all cases, the shear associated with the inflection points would have been reduced. Since the inflectional profiles are all inviscidly unstable with growth rates proportional to the shear, the resulting instabilities would be weakened by the suction/injection process.

The feasibility of the selective suction as a drag-reducing concept has been demonstrated by Gad-el-Hak and Blackwelder.[54] Low-speed streaks were artificially generated in a laminar boundary layer using the method of Gad-el-Hak and Hussain,[56] and a hot-film probe was used to record the near-wall signature of the streaks. An equivalent suction coefficient of $C_q = 0.0006$ was sufficient to eliminate the artificial events and prevent bursting. This rate is five times smaller than the asymptotic suction coefficient for a corresponding turbulent boundary layer. If this result is sustained in a naturally developing turbulent boundary layer, a skin-friction reduction of close to 60% would be attained. Gad-el-Hak and Blackwelder[54] propose to combine suction with nonplanar surface modifications. Minute longitudinal roughness elements if properly spaced in the spanwise direction greatly reduce the spatial randomness of the low-speed streaks (Johansen and Smith[73]). By withdrawing the streaks forming near the peaks of the roughness elements, less suction should be required to achieve an asymptotic boundary layer. Recent experiments by Wilkinson and Lazos[150] and Wilkinson[147] combine suction/blowing with thin-element riblets. Although no net drag reduction is yet attained in these experiments, their results indicate some advantage of combining suction with riblets as proposed by Gad-el-Hak and Blackwelder.[53,54] The recent numerical experiments of Choi et al.[28] also validate the concept of targeting suction/injection to specific near-wall events. Their direct numerical simulations indicate a 20% net drag reduction accompanied by significant suppression of the near-wall structures and the Reynolds stress throughout the entire wall-bounded flow.

Two new developments have relevance to the issue at hand. First, the recently demonstrated ability to revert a chaotic system to a periodic one may provide optimal nonlinear control strategies for further reduction in the amount of suction (or the energy expenditure of any other active wall-modulation technique) needed to attain a given degree of flow stabilization. This is important since, as seen from Eq. (1), net drag reduction achieved in a turbulent boundary layer increases as the suction coefficient decreases. Second, to selectively remove the randomly occurring low-speed streaks, for example, would ultimately require interactive control. In that case, an event is targeted, sensed, and subsequently modulated. Microfabrication technology provides opportunities for practical implementation of the required large array of inexpensive, programmable sensor/actuator chips. Both of these novel developments will be discussed in the following two sections.

V. Control of Chaos

A. Prologue

In the theory of dynamical systems, the so-called *butterfly effect* (the notion that a butterfly stirring the air today in one location could transform future storm systems thousands of kilometers away) denotes sensitive dependence of nonlinear differential equations on initial conditions. The solution of such equations may be in the form of a strange attractor whose intrinsic structure contains a well-defined mechanism to produce a chaotic behavior without requiring random forcing. A question arises naturally: just as small disturbances can radically grow within a deterministic system to yield rich, unpredictable behavior, could minute adjustments to a system parameter be used to reverse the process and control the behavior of a chaotic system? Quite recently, that question was answered in the affirmative theoretically as well as experimentally, at least for system orbits that reside on low-dimensional strange attractors. Scientists at the University of Maryland proposed a control mechanism that was used at the Naval Surface Warfare Center to control the chaotic behavior of a physical system. By making *small* time-dependent adjustments to one of the parameters governing the system's behavior, any one of the infinite number of unstable orbits naturally occuring in the chaotic attractor could be targeted and stabilized. Other desirable characteristics of this control strategy for our application are that the dynamical system need not be analytically known and systems governed by partial differential equations can also be covered, provided their chaotic motions are sufficiently low dimensional.

In the following subsection, the recent attempts to construct a low-dimensional dynamical system approximation for turbulent boundary layers are summarized. Several contemporary strategies for controlling chaotic systems are briefly described in Sec. V.C.

B. Dynamical Systems Approximation

In recent years many nonlinear dynamical systems, both conservative as well as dissipative, have been found to have extreme sensitivity to initial conditions and system parameters. The resulting chaotic behavior and the different bifurcations that could lead to it have been under close scrutiny. In some of the simpler physical systems it has been possible to find a correlation between theory and experiment (see, for example, Sen et al.[122] and Gorman et al.[59]). Other problems, particularly those with infinite degrees of freedom, are still not susceptible to an easy dynamical systems approximation. Boundary-layer turbulence, described by a set of nonlinear partial differential equations, belongs to this category. In one significant attempt, the so-called proper orthogonal, or Karhunen-Loève, decomposition method (Lumley[93–95] and Sirovich[132]) has been used to extract a low-dimensional dynamical system from experimental data of the wall region (Aubry et al.[5] and Aubry[4]). The notion that a complex, infinite-dimensional flow can be decomposed into several low-dimensional subunits is a natural consequence of the realization that quasiperiodic coherent structures dominate the dynamics of seemingly random turbulent shear flows. This implies that low-dimensional, localized dynamics can exist in formally infinite-dimensional extended systems—such as open turbulent flows.

Aubry et al.[5] expanded the instantaneous velocity field of a turbulent boundary layer using experimentally determined eigenfunctions that are in the form of streamwise rolls. They expanded the Navier-Stokes equations using these eigenfunctions, applied a Galerkin projection, and then truncated the infinite-dimensional representation to obtain a 10-dimensional set of ordinary differential equations. Reducing the physics to a finite-dimensional dynamical system enables a study of its behavior through an examination of the fixed points and the topology of their stable and unstable manifolds. The equations represent the dynamical behavior of the rolls and are shown to exhibit a chaotic regime as well as an intermittency due to a burstlike phenomenon. However, Aubry et al.'s 10-mode dynamical system displays a *regular* intermittency, in contrast both to that in actual turbulence as well as to the chaotic intermittency encountered by Pomeau and Manneville[101] in which event durations are distributed stochastically. Nevertheless, the major conclusion of Aubry et al.'s study is that the bursts appear to be produced autonomously by the wall region even without turbulence but are triggered by turbulent pressure signals from the outer layer.

More recently, Berkooz et al.[12] generalized the class of wall-layer models developed by Aubry et al.[5] to permit uncoupled evolution of streamwise and cross-stream disturbances. Berkooz et al.'s results suggest that the intermittent events observed in Aubry et al. do not arise solely because of the effective closure assumption incorporated but are rather rooted deeper in the dynamical phenomena of the wall region.

In addition to the reductionist viewpoint exemplified by the work of Aubry et al.[5] and Berkooz et al.,[22] attempts have been made to determine directly the dimension of the attractors underlying specific turbulent flows. The central issue here is whether or not turbulent solutions to the infinite-dimensional Navier-Stokes equations can be asymptotically described by a finite number of degrees of freedom. Grappin and Léorat[60] computed the Lyapunov exponents and the attractor dimensions of two- and three-dimensional periodic turbulent flows without shear. They found that the number of degrees of freedom contained in the large scales establishes an upper bound for the dimension of the attractor. Deane and Sirovich[32] and Sirovich and Deane[133] numerically determined the number of dimensions needed to specify chaotic Rayleigh-Bénard convection over a moderate range of Rayleigh numbers *Ra*. They suggested that the *intrinsic* attractor dimension is $\mathcal{O}[Ra^{2/3}]$.

The corresponding dimension in wall-bounded flows appears to be dauntingly high. Keefe et al.[76] determined the dimension of the attractor underlying turbulent Poiseuille flows with spatially periodic boundry conditions. Using a coarse-grained numerical simulation, they computed a lower bound on the Lyapunov dimension of the attractor to be approximately 352 at a pressure-gradient Reynolds number of 3.2×10^3. Keefe et al.[76] argue that the attractor dimension in fully resolved turbulence is unlikely to be much larger than 780. This suggests that periodic turbulent shear flows are deterministic chaos and that a strange attractor does underlie solutions to the Navier-Stokes equations. Temporal unpredictability in the turbulent Poiseuille flow is thus due to the exponential spreading property of such attractors. Although finite, the computed dimension invalidates the notion that the global turbulence can be attributed to the interaction of a *few* degrees of freedom. Moreover, in a physical channel or boundary layer, the flow is not periodic and is open. The attractor dimension in such a case is not known but is believed to be even higher than the estimate provided by Keefe et al. for the periodic (*quasiclosed*) flow.

In contrast to closed, absolutely unstable flows, such as Taylor-Couette systems, where the number of degrees of freedom can be small, local measurements in open, convectively unstable flows, such as boundary layers, do not express the global dynamics, and the attractor dimension in that case may inevitably be too large to be determined experimentally. According to the estimate provided by Keefe et al.,[76] the colossal data required (about 10^D, where D is the attractor dimension) for measuring the dimension simply exceeds current computer capabilities.

C. Chaos Control

There is another question of greater relevance here. Given a dynamical system in the chaotic regime, is it possible to stabilize its behavior through some kind of active control? This is of importance in problems such as chaos in national telephone lines, electric power grids, computer local area networks (Huberman[65]), and turbulent boundary layers. Obvious procedures such as stabilization through *finite* change of parameters to remove the dynamics out of the chaotic regime may often be impractical. In the language of turbulence this is equivalent to relaminarization through, for example, reduction in the Reynolds number or massive wall suction. In this subsection, several recent attempts at controlling nonlinear systems are briefly reviewed. To apply any of the proposed strategies to the intermittent bursting events occurring in the complex, infinite-dimensional turbulent boundary layer is highly problematic, however, and much research work is still needed. We will return to this point in Secs. VII.C and VII.D.

Although other alternatives have been devised (Fowler,[47] Hübler and Lüscher,[67] and Huberman and Lumer[66]), the recent method proposed by workers at the University of Maryland (Ott et al.,[104,105] Shinbrot et al.,[126–130] and Romerias et al.[119]) promises to be a significant breakthrough (see also the two reviews by Shinbrot[123] and Shinbrot et al.[128]). Yorke and his colleagues demonstrated, through numerical experiments with the Henon map, that it is possible to stabilize a chaotic motion about any prechosen, unstable orbit through the use of relatively small perturbations.

Ott-Grebogi-Yorke's (OGY) control strategy has several advantages that are of special concern in the control of turbulence: the mathematical model for the dynamical system need not be known, and only *small* changes in the control parameter are required. The procedure consists of applying minute time-dependent perturbations to one of the system parameters to control the chaotic system around one of its many unstable periodic orbits. In this context, *targeting* refers to the process whereby an arbitrary initial condition on a chaotic attractor is steered toward a prescribed point (target) on this attractor. The goal is to reach the target as quickly as possible using a sequence of small perturbations (Kostelich et al.[78]).

The success of the OGY strategy for controlling chaos hinges on the fact that beneath its apparent unpredictability lies an intricate but highly ordered structure. Left to its own recourse, a chaotic system continually shifts from one periodic pattern to another, creating the appearance of randomness. An appropriately controlled system, on the other hand, is locked into one particular type of repeating motion. With such an active control the dynamical system becomes one with a stable behavior.

The OGY method has been successfully applied in a relatively simple experiment by Ditto et al.[34] and Ditto and Pecora[33] at the Naval Surface Warfare Center, in which *reverse chaos* was obtained in a parametrically driven, gravitationally buckled, amorphous magnetoelastic ribbon. This achievement was significant enough to have been noted in the popular, scientific and otherwise, press (Peterson,[106] Langreth,[86] Corcoran,[30] and Begley[9]). Garfinkel et al.[58] applied the same control strategy to stabilize drug-induced cardiac arrhythmias in sections of a rabbit ventricle.

Other extensions, improvements and applications of the OGY strategy include higher dimensional targeting (Auerbach et al.,[7] Kostelich et al.[79]), controlling chaotic scattering in Hamiltonian (i.e., nondissipative, area conservative) systems (Lai et al.[83,85]), synchronization of identical chaotic systems that govern communication, neural or biological processes (Lai and Grebogi[82]), use of chaos to transmit information (Hayes et al.[62]), control of transient chaos (Lai et al.[84]), and taming spatio-temporal chaos using a sparse array of controllers (Chen et al.,[27] Qin et al.,[116] and Auerbach[6]).

Very recently, Keefe[74,75] made a useful comparison between two nonlinear control strategies as applied to fluid problems: Ott-Grebogi-Yorke's feedback method described earlier and the model-based control strategy originated by Hübler (see, for example, Hübler and Lüscher[67] and Lüscher and Hübler[97]), the H-method. The former strategy exploits the sensitivity of chaotic systems to stabilize existing periodic orbits and steady states. Some feedback is needed to steer the trajectories toward the chosen fixed point, but the required control signal is minuscule. In contrast, Hübler's scheme does not explicitly make use of the system sensitivity. It produces general control response (periodic or aperiodic) and needs little or no feedback, but its control inputs are generally large.

Keefe[74] first examined numerically the two schemes as applied to fully developed and transitional solutions of the Ginzburg-Landau equation, an evolution equation that governs the initially weakly nonlinear stages of transition in several flows and that possesses both transitional and fully chaotic solutions. The Ginzburg-Landau equation has solutions that display either absolute or convective instabilities and is thus a reasonable model for both closed and open flows. Keefe's main conclusion is that control of nonlinear systems is best obtained by making maximum use possible of the underlying natural dynamics. If the goal dynamics is an unstable nonlinear solution of the equation and the flow is nearby at the instant control is applied, both methods perform reliably and at low-energy cost in reaching and maintaining this goal. Predictably, the performance of both control strategies degrades due to noise and the spatially discrete nature of realistic forcing.

Subsequently, Keefe[75] repeated the numerical experiment in an attempt to reduce the drag in a channel flow with spatially periodic boundary conditions. The OGY method reduces the skin friction to 60–80% of the turbulent value at a mass-flux Reynolds number of 4.408×10^3. The H-method fails to achieve any drag reduction when starting from fully turbulent initial condition but shows potential for suppressing or retarding laminar-to-turbulent transition. Keefe[74] suggests that the H-strategy might be more appropriate for boundary-layer control, whereas the OGY method might best be used for channel flows.

It is also relevant to note here the work of Bau and his colleagues at the University of Pennsylvania (Singer et al.[131] and Wang et al.[146]), who could devise a feedback control to stabilize (*relaminarize*) the naturally occurring chaotic oscillations of a toroidal thermal convection loop heated from below and cooled from above. Based on a simple mathematical model for the *thermosyphon,* Bau and his colleagues constructed an interactive control system that was used to alter significantly the flow characteristics inside the convection loop. Their linear control strategy, perhaps a special version of the OGY's feedback method, consists simply of sensing the deviation of fluid temperatures from desired values at a number of locations inside the thermosyphon loop and then altering the wall heating either to suppress or to enhance such deviations. Wang et al.[146] also suggested extending their theoretical and experimental method to more complex situations such as those involving Bénard convection (Tang and Ban[140,141]). Hu and Bau[64] used a similar feedback control strategy to demonstrate that the critical Reynolds number for the loss of stability of planar Poiseuille flow could be significantly increased or decreased.

Several other control strategies have become available during 1993, each with its own advantages and potential pitfalls. With a long-range objective of developing effective methods for the control of turbulent flows, Choi et al.[29] have recently applied the optimal control theory to the stochastically forced Burgers equation. Giving the complexity of a real turbulent flow and the lack of complete physical understanding, Choi et al.'s[29] strategy is to appeal to the more systematic but less intuitive methods of control theory. They offer a *suboptimal* control and feedback procedure for general stationary and time-dependent problems using methods of calculus of variations through the adjoint state and gradient algorithms. With the primary goal of cost minimization, Choi et al.[29] investigated both distributed and boundary controls of the nonlinear Burgers equation. In a subsequent paper, Bewley et al.[14] extended the technique to control a numerically generated turbulent channel flow. Again, the scheme provides substantial decrease in both the cost function and the overall drag with reasonable levels of control input.

Berkooz et al.[12] applied techniques of modern control theory to estimate the phase-space location of dynamical models of the wall-layer coherent structures and used these estimates to control the model dynamics. Since discrete wall sensors provide incomplete knowledge of phase-space location, Berkooz et al. maintain that a nonlinear observer, which incorporates past information and the equations of motion into the estimation procedure, is required. Using an extended Kalman filter, they achieved effective control of a bursting pair of rolls with the equivalent of two wall-mounted shear sensors.

Shinbrot and Ottino[124,125] offer yet another strategy presumably most suited for controlling coherent structures in area-preserving turbulent flows. Their geometric method exploits the premise that the dynamical mechanisms that produce the organized structures can be remarkably simple. By repeated stretching and folding of "horseshoes" that are present in chaotic systems, Shinbrot and Ottino have demonstrated numerically as well as experimentally the ability to create, destroy, and manipulate coherent structures in chaotic fluid systems. The key idea to create such structures is to intentionally place folds of horseshoes near low-order periodic points. Shinbrot and Ottino[124] applied the technique to three prototypical problems: a one-dimensional chaotic map, a two-dimensional one, and a chaotically advected fluid.

VI. Microfabrication

Manufacturing processes that can create microscopic machinery are termed microfabrications. In this emerging technology, under intensive development only since 1990, electronic and mechanical components are combined on a single silicon chip (newer chips are not necessarily silicon based, and a variety of other materials are continuously being introduced). Sensors for pressure, temperature, velocity, mass flow, or sound are currently combined with motors, electrostatic actuators, pneumatic actuators, valves, gears, or tweezers on single programmable elements of typical size $\mathcal{O}[10\,\mu]$.

Microelectromechanical systems (or MEMS) have been constructed and tested during the last few years at, among others, Massachusetts Institute of Technology, AT&T Bell Laboratories, California Institute of Technology, University of California at Berkeley, University of California at Los Angeles, Honeywell Corporation, University of Michigan, Case Western University, University of Minnesota, Robert Bosch GmbH (Germany), University of Tokyo, Toyota Central R&D Laboratories, and Chalmers University of Technology (Sweden).

The *Journal of Microelectromechanical Systems* is devoted to this new technology. Entire sessions in scientific meetings have been increasingly assigned to MEMS applications in fluid mechanics (see, for example, the presentations by McMichael,[101] Tai,[138] Mehregany,[102] Mastrangelo,[98] and Yun,[154] all made at the AIAA 3rd Shear Flow Control Conference, Orlando, Florida, July 6–9, 1993). For the reader interested in the intricate details of current research, see, for example, the papers by Behi et al.,[10] Gabriel et al.,[48,49] Pister et al.,[109] Döring et al.,[35] Pister,[108] Breuer,[18] Ho,[63] Lipkin,[91] Mastrangelo,[109] Mastrangelo and Hsu,[99,100] and Tai.[139] Lipkin[91] briefly describes the recent development of a *microsteam engine* designed and built by J. J. Sniegowski of Sandia National Laboratories.

MEMS would be ideal for the interactive flow control concept advocated in the present paper. Methods of flow control targeted toward specific coherent structures involve nonintrusive detection and subsequent modulation of events that occur randomly in space and time. To achieve proper targeted control of these quasiperiodic vortical events, temporal phasing as well as spatial selectivity are required. Practical implementation of such an idea necessitates the use of a large number of intelligent, interactive wall sensors and actuators arranged in a checkerboard pattern.

The sensors would be expected to measure the amplitude, location, and phase or frequency of the signals impressed upon the wall by incipient bursting events. Instantaneous wall-pressure or wall-shear stress could be sensed, for example. The normal or in-plane motion of a minute membrane is proportional to the respective point force of primary interest. For measuring wall pressure, microphonelike devices respond to the motion of a vibrating surface membrane or an internal elastomer. Several types are available including piezoelectric, variable-capacitance (condenser or electret), ultrasonic, and optical (e.g., optical-fiber and diode-laser) devices. A potentially useful technique for our purposes has been recently tried at MIT (J. H. Haritonidis, private communications). An array of extremely small (0.2 mm in diameter) laser-powered microphones (termed picophones) was machined in silicon using integrated circuit fabrication techniques and was used for field measurement of the instantaneous surface pressure in a turbulent boundary layer.

Actuators are expected to produce a desired change in the targeted coherent structures. The local acceleration action needed to stabilize an incipient bursting event could be in the form of adaptive wall, transpiration, or wall heat transfer. Traveling surface waves may be used to modify a locally convecting pressure gradient such that the wall motion follows that of the coherent event causing the pressure change. Surface motion in the form of a Gaussian hill with height $y^+ = \mathcal{O}[10]$ should be sufficient to suppress typical incipient bursts (Lumley[96]). Such time-dependent alteration in wall geometry could be generated by driving a flexible skin using an array of piezoelectric devices (dilate or contract depending on the polarity of current passing through them), electromagnetic actuators, magnetoelastic ribbons (made of nonlinear materials that change their stiffness in the presence of varying magnetic fields), or Terfenol-d rods (a novel metal composite, developed at Grumman Corporation, which changes its length when subjected to a magnetic field). Note should also be made of other exotic materials that might be used for actuation. For example, electrorheological fluids (Halsey and Martin[61]) instantly solidify when exposed to an electric field and might thus be useful for the present application.

Suction/injection at many discrete points may be achieved by simply connecting a large number of minute streamwise slots, arranged in a checkerboard pattern, to a low-pressure/high-pressure reservoir located under the working surface. The transpiration through each individual slot is turned on and off using a corresponding number of independently controlled microvalves. Based on the results of Gad-el-Hak and Blackwelder,[54] summarized in Sec. IV.B, equivalent suction coefficients of about 0.0006 should be sufficient to stabilize the near-wall region. Assuming that the skin friction in the uncontrolled boundary layer is $C_f = 0.003$, and assuming further that the suction used is sufficient to establish an asymptotic boundary layer ($d\theta/dx = 0$), the skin friction coefficient in the interactively controlled case is then $C_f = 0 + 2C_q = 0.0012$ or 40% of the original value. The net benefit will, of course, be reduced by the energy expenditure of the suction pump as well as the array of microsensors and microvalves.

Finally, if the bursting events are to be eliminated by lowering the near-wall viscosity, direct electric-resistance heating could be used in liquid flows and thermoelectric devices based on the Peltier effect (Soo[134]) could be used for cooling in the case of gaseous boundary layers. The absolute viscosity of water at 20°C decreases by approximately 2% for each 1°C rise in temperature, where for room-temperature air, μ decreases by approximately 0.2% for each 1°C drop in temperature. Equation (3) can be used to show that a suction coefficient of 0.0006 has approximately the same effect on the curvature of the instantaneous velocity profile at the wall as a surface heating of 2°C in water or a surface cooling of 40°C in air (Liepmann and Nosenchuck[89] and Liepmann et al.[90]).

Sensors and actuators of the types discussed earlier could be combined on individual electronic chips using microfabrication technology. The chips could be interconnected in a communication network that is controlled by a massively parallel computer or a self-learning neural network. Microfabrication could in the future lead to the production of large arrays of small, programmable sensors/actuators at low prices.

Factors to be considered in an eventual field application of chips produced using microfabrication processes include sensitivity of sensors, sufficiency and frequency response of actuators' action, fabrication of large arrays at affordable prices, survivability in the adverse field environment, and energy required to power the sensors/actuators. As will be shown in the two examples in Sec. VII.B, sensor/actuator chips currently produced are small enough for typical field application, and they could be programmed to provide a given action in response to a certain sensor output. Present prototypes are, however, still quite expensive as well as delicate. But so was the transistor when first introduced! It is hoped that the unit price of future sensor/actuator elements would follow the same dramatic trends witnessed in the case of the simple transistor and even the much more complex integrated circuit. Field applications of sensor/actuator arrays also necessitate a high degree of durability and a modest energy consumption.

VII. Futuristic Flow Control Systems

A. Introductory Remarks

As was discussed in Sec. II.C, the global suction coefficient required to achieve a zero-growth turbulent boundary layer is typically $C_q \approx 0.003$. (This value changes with the Reynolds number and so does the skin-friction coefficient.) This amount is far too large to yield net skin-friction reduction. The question of relevance here is whether or not targeting and modifying specific structures in the flow would lead to the same saving (i.e., $d\theta/dx = 0$) but with substantially reduced penalty (i.e., smaller C_q). The results of the selective suction experiment presented in Sec. IV.B indicate a positive answer. Furthermore, an even more impressive reduction in the suction coefficient is potentially possible if one is to employ any of the chaos control strategies described in Sec. V.C.

In Lumley's[96] view, a bursting event corresponds to a dynamical system leaving one fixed point and jumping to another along a heteroclinic cycle. Delaying this jump by holding the system near the first fixed point should lead to lower momentum transport in the wall region and, therefore, to lower skin-friction drag. Practical applications of methods targeted at controlling a particular turbulent structure to achieve a prescribed goal would perhaps require implementing a large number of surface sensors/actuators together with appropriate open (or closed, feedback) loops and control algorithms. The emerging field of microfabrication, where microscopic machinery are fashioned on a single silicon chip, may in the future be utilized to realize such *smart* arrays at reasonable cost. Additionally, newly formulated theories for controlling nonlinear dynamical systems could be used to optimize the relevant processes.

This section is organized into four subsections. Following the present introductory remarks, two examples from typical field applications are used to illustrate the required dimension, frequency response, and number of sensor/actuator elements. In Sec. VII.C, a road map and a blueprint are made to help establish a control theory for intermittent chaotic systems. Finally, Sec. VII.D describes some of the potential pitfalls associated with eventual application of the idea advocated herein to field situations.

B. Required Chracteristics

It is instructive to estimate some representative characteristics of the required array of sensors/actuators. Consider a typical commercial aircraft cruising at a speed of 300 m/s and at an altitude of 10 km. The kinematic viscosity of air and the unit Reynolds number in this case are, respectively, $\nu = 0.3$ cm^2/s and $Re = 10^7$/m. Assume further that the portion of fuselage to be controlled has turbulent boundary-layer characteristics that are identical to those for a zero-pressure-gradient flat plate at a distance of 1 m from the leading edge. In this case, the skin-friction coefficient and the friction velocity are, respectively, $C_f = 0.003$ and $u^* = 1162$ cm/s. At this location, one viscous wall unit is only $\nu/u^* = 2.6\ \mu$. For the surface array of sensors/actuators to be hydraulically smooth, it should not protrude beyond the viscous sublayer, or $5\nu/u^* = 13\ \mu$.

Wall-speed streaks are the most visible, reliable, and detectable indicators of the preburst turbulence production process. The detection criterion is simply low velocity near the wall, and the actuator response should be to accelerate (or to remove) the low-speed region before it breaks down. Local wall motion, suction, tangential injection, or heating triggered on sensed wall-pressure or wall-shear stress could be used to cause local acceleration of near-wall fluid.

The recent numerical experiments of Berkooz et al.[12] indicate that effective control of bursting pair of rolls may be achieved by using the equivalent of two wall-mounted shear sensors. If the goal is to stabilize or to eliminate *all* low-speed streaks in the boundary layer, a reasonable estimate for the spanwise and streamwise distances between individual elements of a checkerboard array is, respectively, 100 and 1000 wall units (these are equal to, respectively, the average spanwise wavelength between two adjacent streaks and the average streamwise extent for a typical low-speed region), or 260 and 2600 μ, for our particular example. A reasonable size for each element is probably one-tenth of the spanwise separation, or 26 μ. A 1 m $\times$ 1 m portion of the surface would have to be covered with about 1.5×10^6 elements. This is a colossal number, but the density of sensors/actuators could be considerably reduced if we moderate our goal of targeting every single bursting event.

It is well known that not every low-speed streak leads to a burst. On the average, a particular sensor would detect an incipient bursting event every wall-unit interval of $P^+ = Pu^{*2}/\nu = 250$, or $P = 56\ \mu$s . The corresponding dimensionless and dimensional frequencies are $f^+ = 0.004$ and $f = 18$ kHz, respectively. At different distances from the leading edge and in the presence of nonzero-pressure gradient, the sensors/actuators array would have different characteristics, but the corresponding numbers would still be in the same ballpark as estimated here.

As a second example, consider an underwater vehicle moving at a speed of 10 m/s. Despite the relatively low speed, the unit Reynolds number is still the same as estimated earlier for the air case, $Re = 10^7$/m , due to the much lower kinematic viscosity of water. At 1 m from the leading edge of an imaginary flat plate towed in water at the same speed, the friction velocity is only $u^* = 39$ cm/s, but the wall unit is still the same as in the aircraft example, $\nu/u^* = 2.6\ \mu$. The density of the required sensors/actuators array is the same as computed for the aircraft example, 1.5×10^6 elements/m^2. The anticipated average frequency of sensing a bursting event is, however, much lower at $f = 600$ Hz .

As computed in the two preceding examples, both the required size for a sensor/actuator element and the average frequency at which an element would be activated are within the currently known capabilities of microfabrication technology. The number of elements needed per unit area is, however, alarmingly large. The unit cost of manufacturing a programmable sensor/actuator element would have to come down dramatically, perhaps matching the unit cost of a conventional transistor, before the idea advocated herein would become practical.

A second consideration is the energy consumed by each sensor/actuator element. Total energy consumption by the entire control system obviously has to be low enough to achieve net savings. Current prototypes do not satisfy this criterion (Reynolds[117]), however, and further advances in this area are needed. Consider the following calculations for the aircraft example. One meter from the leading edge, the skin-friction drag to be reduced is approximately 150 N/m^2. Engine power needed to overcome this retarding force per unit area is 4.5×10^4 W/m^2. To achieve net savings, energy consumption by each sensor/actuator element has, therefore, to be substantially less than 0.03 W. At present, a typical microactuator requires about 0.1 W to do useful work. For a 60% drag-reduction goal (a not-too-farfetched goal according to the selective suction results discussed in Sec. IV.B), this energy consumption has to come down to 0.018 W just to break even. For future microsensors/microactuators, a reasonable energy-consumption target to aim for should perhaps be 0.01 W/element or less.

In either the airplane or the submarine case, the actuator's response need not be too large. As shown in Sec. VI, wall displacement on the order of 10 wall units (26 μ in both examples), suction coefficient of about 0.0006, or surface cooling/heating on the order of 40°C /2°C (in the first/second example, respectively) should be sufficient to stabilize the turbulent flow.

Similar calculations have been made recently by Reynolds[117] and Wadsworth et al.[145] Their results agree closely with the estimates made in the present paper for typical field requirements.

C. Control of Intermittent Chaotic Systems

Recent advances in dynamical systems theory provide opportunities for achieving efficient control of turbulent boundary layers. As was shown in Sec. V.C, many control strategies are available for nonlinear systems. None is straightforward or without potential pitfalls, and much research is needed before practical implementation to the complex, infinite-dimensional turbulent wall-bounded flow. For the present purposes, we focus on Ott-Grebogi-Yorke's feedback method, but the arguments presented here could be extended to other control schemes.

Researchers at the University of Maryland and the Naval Surface Warfare Center have successfully controlled the chaotic behavior of a one-degree-of-freedom physical system, whose orbits reside on a strange attractor, by making small time-dependent adjustments to one of the parameters governing the system's behavior. In the control scheme envisaged here, the control theory developed at the University of Maryland is to be extended to include intermittent systems. The sequence of quasiperiodic events in the wall region of a turbulent boundary layer is to be modeled as a dynamical system. Although helpful, it should be noted that knowledge of the dynamical system is not required for the application of the control method but is merely used to test the theory. In the following, a *road map* and a *blueprint* are made to help establish the control theory for intermittent dynamical systems.

The boundary-layer bursting phenomenon is intermittent, a term that refers to the occurrence of a signal that alternates randomly between long regular phases and relatively short irregular bursts. Such signals have also been observed in other diverse physical experiments: Rayleigh-Bénard convection (Bergé et al.[11]); nonlinear oscillators (Jeffries and Pérez[72]), Belousov-Zhabotinsky reaction (Pomeau et al.[111]), and Josephson junctions (Yeh and Kao[153]). The common denominator of all of these intermittent systems is that their signals, consisting of discrete time values, can be represented by one of three types of Poincaré sections that result in the intermittent systems defined by Pomeau and Manneville.[110] What distinguishes these three types of intermittencies is whether the eigenvalues of their Poincaré maps (at any particular point) cross the unit circle at ±1 or as complex conjugates. We note that even though in all three cases a single unstable fixed point is illustrated, more than one such point may exist for a fixed set of parameter values. This indeed appears to be the case relevant to the boundary layer intermittency. Aubry et al.[5] note that the intermittency in their low-dimensional dynamical system is a heteroclinic cycle corresponding to a type II intermittency (complex conjugates crossings) with two unstable fixed points.

The control method to be used is based upon a push toward a fixed point in phase space. The procedure for controlling boundary layer intermittency will be such as to first construct a Poincaré map to classify the type of intermittency. Then we apply a *small* time-dependent perturbation to the parameter controlling the stability of the fixed point. To be more concrete, assume that we have constructed such a Poincaré map:

$$x'_{i+1} = f(x_i, p) \tag{4}$$

where x_i is the discrete time signal, and p is the parameter we want to control. Let x_* denote the unstable fixed point of the map f at the nominal parameter value p_*. If we now apply a small time-dependent perturbation to this parameter, the following first-order approximation of the perturbed map results:

$$x'_{i+1} \approx a x'_i + b p'_i \tag{5}$$

where

$$x'_i = x_i - x_*, \qquad p'_i = p_i - p_* \tag{6}$$

and

$$a = \left[\frac{\mathrm{d}f}{\mathrm{d}x}\right]_{x_*, p_*}, \qquad b = \left[\frac{\mathrm{d}f}{\mathrm{d}p}\right]_{x_*, p_*} \tag{7}$$

Note that we have taken $p = p_i$. Now the control mechanism consists of finding a linear control law:

$$p'_i = k x'_i \tag{8}$$

such that the eigenvalue of $a + bk$ remains within the circle of stability.

We note that, for the control to take effect, the trajectory of the system must come close to the unstable fixed point (i.e., within the circle of stability). This is guaranteed to happen within the mean intermittency time of the signal. Once within this circle, time-dependent control is activated guaranteeing that the orbit will not leave. This guarantee can only be made if no noise is present in the system. Since noise will always be present (due to the background turbulence) and will tend to *kick* the orbit out of the circle, then the only way to keep the orbit close to the unstable fixed point is to increase the control amplitude to overcome the detrimental effect of noise.

A logical way to proceed before an experimental demonstration is first to apply the envisaged control to a series of theoretical cases, each one being slightly more complicated than the previous one, but bringing in more of the physics relevant to a turbulent boundary layer. In each case a control parameter is to be selected and the technique adapted to meet the special demands of the model's characteristics under study. The following sequence of model problems may serve to develop and test the control algorithm that is sought:

1) Lorenz model (Lorenz[92] and Sparrow[135]): This is a simplified model of cellular convection (Gorman et al.[59]). It is well studied and very simple, has a chaotic attractor, and is an obvious starting point. This problem has also been discussed by Wang et al.[146] An infinite set of ordinary differential equations was needed to describe the dynamics of a simple convection loop, but three of these equations, similar to the celebrated Lorenz equations, decoupled from the set with exact closure and could be solved independently without the need for truncation.

2) Intermittency models: Several types of intermittency have been identified by Pomeau and Manneville.[110] A new theory of control is to be developed for application to random bursting events that occur in these models.

3) Wall-region model (Aubry et al.[5]): This is the 10-mode dynamical system discussed in Sec. V.B. The aim is to control the bursting events superposed on the fixed point solution of the model. The control parameter is to be chosen from the point of view of effectiveness as well as its relation to some physical quantity within the turbulent boundary layer that can eventually be acted upon.

4) Langmuir instability model (Leibovich and Paolucci[88] and Phillips[107]): This model has been used in the study of Langmuir circulation in oceans and lakes and has recently been noted to be applicable to the turbulent boundary layer instability leading to the formation of streamwise vortices in the wall region.

5) Other models: Direct numerical simulation, experimental data, and simplified modeling of the physical bursting process is to be used to obtain other systems to be analyzed. Development of these models is to take into account the fact that their responses to the proposed control variable should approximately parallel that of an actual boundary layer. Previous models were not really developed for control purposes but rather for analysis and physical understanding. The structural stability of the new control system to bursting events described by these models is also to be tested.

D. Potential Pitfalls

The idea advocated in the preceding subsection is to exploit the exponential sensitivity of a chaotic system to minute perturbations to direct the system to a desired accessible state in a short time (the OGY control strategy). The near-wall events in a turbulent boundary layer are to be modeled as a nonlinear dynamical system. Efficient control about any one of the system's unstable fixed points (periodic orbits) embedded in the chaotic attractor is to be effected. In such a scheme, sensors detect incipient bursting events (the continuous, and random, tendency of the nonlinear dynamical system to leave a fixed point in phase space and migrate toward another fixed point), and actuators respond by *pushing* the system back toward the first fixed point. There are, however, some potential pitfalls with the proposed scenario. For example, *noise* in the system, incoherent turbulence in our case, tends to kick the orbit out of the circle of stability that surrounds the unstable fixed point. This would force the operator to increase the control amplitude to keep the orbit close to the fixed point—undesired but not necessarily fatal.

Systems with an infinite number of degrees of freedom, such as turbulent boundary layers, are not readily susceptible to an easy dynamical systems approximation. It should be noted, however, that the control theory developed by Ott et al.[104,105] for nonintermittent systems does not require precise analytical knowledge of the dynamical system, provided a Poincaré map is available. We will return to this point at the end of this subsection.

For practical implementation of the concept advocated in this paper, a large, but finite, number of sensor/actuator elements is to be arranged on a checkerboard. The spatially distributed control is most conveniently located at the wall and not in the interior of the boundary layer. This means that sensed information is incomplete and might be misinterpreted by the sensors. Although some bursts might not be sensed altogether, some others would be detected but the response might not be in the appropriate location. Additionally, the discrete system might lead to stray control signals that could inadvertently destabilize some low-speed streaks which would have otherwise remained stable (recall from Sec. 7.2 that in an uncontrolled flow, many streaks simply whither away without leading to bursts). As a result, the checkerboard actuators might be less effective, and the performance of the control scheme might degrade. An additional consideration is the potential for an adverse mutual interaction, via the altered flow, between the different elements of a large array.

Lumley[96] points to yet another potential pitfall. Implicit in the control scheme is the assumption that an incipient burst would tend to leave the fixed point along an *average* path. The actuator would push back that burst along the same path. In reality, however, most bursts would be to one side or the other of the average path, which again might lead to lower actuator's effectiveness. A potentially more serious pitfall is that the manifold along which the chaotic system leaves the chosen fixed point might not be one dimensional as implicitly implied in the control theory. It is not clear at present whether this may simply lead to the use of larger control amplitudes or to more serious consequences.

The control scheme proposed by Ott et al.[104] would not function if the number of accessible parameters controlling the underlying dynamics is less than the dimension of the unstable manifold of the orbit whose stability is sought. This condition places severe limitations when trying to control an open system such as a boundary layer whose dimension is potentially quite high (see Keefe et al.[76]).

Two recent advances are relevant to the issue of controlling chaos in high, possibly infinite, dimensional systems. First, attempts have been made to formulate a feedback control that requires modeling the local dynamics of only a single or a few of the possibly infinite number of phase-space variables (Auerbach et al.[7] and Kostelich et al.[79]). Second, Auerbach[6] has recently proposed a modification of the OGY control scheme for taming the dynamics of convectively unstable, spatially extended (open) systems exhibiting chaotic behavior. She has shown how a complex, spatio-temporal phenomena can be eliminated using a sparse array of coupled chaotic elements, in favor of a coherent state in which all elements are synchronized to a prescribed periodic orbit of the uncoupled system.

An alternative expression of this overall challenge of dealing with high-dimensional unstable manifolds is the problem of gaining sufficient knowledge of the underlying dynamics to be able to implement the control. The alternative strategy of experimentally constructing a Poincaré section to characterize the dynamics is attractive but again only for systems whose attractor, strange or otherwise, has low dimension. Systems with even five-dimensional dynamics simply produce scatter plots when a Poincaré section is attempted. In theory, higher-dimensional maps, Poincaré section being a two-dimensional map, may be constructed from experimental data, but this task is far from trivial. For systems with generic nonlinearities, coefficient fitting could alternatively be used for constructing the associated dynamical systems, but this approach is also nontrivial (L. R. Keefe, private communications).

VIII. Concluding Remarks

Numerous methods of flow control are already used in practical engineering devices. Yet, limitations exist for the practical implementation of some known control techniques in specific situations. For example, in attempting to reduce the drag or enhance the lift of a body having a turbulent boundary layer using global suction, the *penalty* associated with the control device often exceeds the *saving* derived from its use. What is needed is a way to reduce this penalty to achieve a more efficient control. The main objective of this article was to illustrate a possible scenario by which more efficient control could be realized. The central idea is to target specific coherent structures for modulation and to exploit the exponential sensitivity of the dynamical system to minimize the control cost.

In this paper, it was argued that future systems for control of turbulent boundary layers would greatly benefit from the merging of a new science and a novel technology. Efficient control of chaotic, nonlinear dynamical systems has been demonstrated theoretically as well as experimentally, at least for simple, low-dimensional systems. Extension of any of the recently available control strategies to high-dimensional dynamical systems such as shear-flow turbulence is not trivial, and much research is needed before practical implementation. However, the potential payoff is enormous and high-risk research is justified.

Microfabrication is an emerging technology that has the potential for producing inexpensive, programmable sensor/actuator chips that have dimensions on the order of a few microns. Together, these two disciplines could provide efficient interactive control systems to achieve a desired beneficial goal for practical flowfields. Such systems are envisaged as consisting of a large number of intelligent, interactive wall sensors and actuators targeted toward specific organized structures that occur randomly within the boundary layer.

Required size and frequency response for the large array of sensor/actuator elements needed for typical field application are within the currently known capabilities of microfabrication technology. Actuator's response would typically be small enough to achieve net saving. Present sensor/actuator prototypes are, however, still quite expensive as well as delicate. Field applications of sensor/actuator arrays necessitate a low unit price, a minuscule energy consumption, and a high degree of durability. All three criteria must be satisfactorily met before such control systems are widely applied.

Finally, the arguments presented here for the goal of achieving skin-friction drag reduction in a turbulent boundary layer could be readily extended to other control goals, such as transition delay, separation postponement, lift enhancement, heat transfer augmentation, and noise suppression, as well as to other flowfields, as for example pipe flows, jets, wakes, and mixing layers. In a field that has not generated many breakthroughs in several decades, the stakes for achieving efficient flow control are enormous and the outlook is optimistic.

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